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On some difficulties with reckoning in the measurement of poverty

by

S. Subramanian

Madras Institute of Development Studies
79, Second Main Road, Gandhi Nagar
Adyar, Chennai 600 020

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AUTHOR'S NAME AND INSTITUTIONAL AFFILIATION

S. Subramanian
Madras Institute of Development Studies
79, II Main Road
Gandhinagar Adyar
Madras 600 020
INDIA

ABSTRACT OF PAPER

This paper points to some elementary conflicts between the claims of individual justice and group justice as they manifest themselves in the process of seeking a real-valued index of poverty which is required to satisfy certain seemingly desirable properties.

ON SOME DIFFICULTIES WITH RECKONING DISTRIBUTIVE JUSTICE IN THE MEASUREMENT OF POVERTY

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S.Subramanian+

Madras Institute of Development Studies

'And no virtue and vice are so much divided as those two virtues'.

- Father Brown

I. MOTIVATION

An issue of potential interest in the measurement of poverty has to do with the way in which poverty is distributed across different well-defined subgroups within the population. Just about the first paper in the formal literature on the measurement of poverty to deal with the relevance of subgroup poverty for an overall assessment of poverty is the one by Foster, Greer and Thorbecke - FGT for short - (1984). In the paper cited, the authors suggest that a desirable property in a poverty index is that of decomposability. Decomposability requires that aggregate poverty in a society be amenable to being expressed as a weighted average of subgroup poverty levels, the weights being the population shares of the subgroups. The decomposability property enables one to infer the proportionate contribution of each subgroup to overall poverty - a datum of great value in the proper designing and targeting of poverty-alleviation schemes. Every decomposable poverty index is also subgroup consistent - subgroup consistency requiring, essentially, that a poverty index should

+ I would like to thank D. Jayaraj and S.Guhan for very many helpful discussions on the subject matter of this paper.

register a decline in value if, everything else remaining the same, poverty in any subgroup declines. Subgroup consistency is a slightly strengthened version of the property which FGT (1984) call 'subgroup monotonicity', and a detailed treatment of this consistency property - including a characterization of the class of subgroup consistent poverty indices - is contained in Foster and Shorrocks (1991).

It is interesting to consider the way in which Foster and Shorrocks motivate their discussion of subgroup consistency (Foster and Shorrocks, 1991; p.687): 'Subgroup consistency may also be regarded as a natural analogue of the monotonicity condition of Sen (1976), since monotonicity requires that aggregate poverty fall ... if one <u>person's</u> poverty is reduced, <u>ceteris paribus</u>, while subgroup consistency demands that aggregate poverty fall if one <u>subgroup's</u> poverty is reduced, <u>ceteris paribus</u>'. In this connection, it is immediately tempting to seek also an analogy between the conventional <u>transfer</u> axiom and a corresponding one which could be devised for subgroups. The transfer - or Pigou-Dalton - condition essentially requires that <u>ceteris paribus</u>, a progressive rank-preserving transfer between two poor individuals should be accompanied by a dimunition in the value of the poverty index. In a similar spirit one could require - speaking loosely for the moment - that aggregate poverty should decline with a move toward equalization of subgroup poverty levels, other things remaining the same. The underlying motivation for this requirement is sought to be made precise in this paper through the postulation of a property I call <u>subgroup sensitivity</u>. Subgroup sensitivity seems to be a natural property to demand of a poverty index in every context wherein our assessment of the nature and extent of poverty is mediated by our concern for the poverty profiles of different subgroups within a population.

For instance, suppose A to be an historically disadvantaged social group, say an ethnic minority in a society; and suppose B - possibly an ethnic majority - to be an historically privileged group. If, other things remaining equal, there is a bridging of the gap in the poverty levels of the two groups over time, then it seems very natural to pronounce that poverty in the society under review has become less pernicious. I submit that a poverty index which registers this value-judgement is a desirable one; only, it turns out that such an index can be purchased only at the cost of having to abandon certain other properties in a poverty index which have for long been held to possess fundamental appeal in the poverty measurement literature. The rest of this paper is devoted to explicating this tension between alternative desirable properties in a poverty index.

2. CONCEPTS AND DEFINITIONS

For every $n=1,2,\ldots,\infty$, let X_n be the set of non-negative vectors $\{(x_1,\ldots,x_i,\ldots,x_n)\}$, and define X to be the set $\cup_{n=1}^{\infty} X_n$. A typical element of the set X is to be interpreted as an income vector x, the typical element x_i of which stands for the ith person's income. The dimensionality of x will be written as n(x). z will stand for the poverty line: it is a positive real number designating a level of income such that any person whose income does not exceed z will be certified to be poor. For all $x \in X$ and all $z \in \mathbb{R}_{++}$, $x^P(x;z)$ will stand for the income vector of the poor population and $x^N(x;z)$ for the income vector of the nonpoor population; and $\mu^P(x;z)$ will denote the average income of the poor in x. Following Foster and Shorrocks (1991), any $x \in X$ will be said to be obtained as a _permutation_ of $y \in X$ if $x = y\pi$ for some permutation matrix π; and $\hat{x}$ will be called the _ordered version_ of x if $\hat{x}$ is derived from x by a permutation for which $\hat{x}_1 \leq \hat{x}_2 \leq \ldots \leq \hat{x}_n$. An income vector $x \in X$ will be said to _vector-dominate_ an income vector $y \in X$ - writen xGy - if $\hat{x}_i \geq \hat{y}_i$ for all i and $\hat{x}_i > \hat{y}_i$ for some i. We shall say that $x \in X$ is derived from $y \in X$ through a _permissible progressive transfer_ if $x_i = y_i$ for all $i \neq j,k$ for some j,k satisfying $y_j < y_k$, $x_j = y_j + \delta$ and $x_k = y_k - \delta$ where $0 < \delta \leq (y_k - y_j)/2$.

A _poverty index_ is a function $P: X \times \mathbb{R}_{++} \rightarrow \mathbb{R}$. Some very fundamental properties which it has been held to be desirable for a poverty index to satisfy are described below.

Definition 2.1: Symmetry (Axiom S). For all $z \in \mathbb{R}_{++}$ and all $x,y \in X$ such that x is derived from y by a permutation, $P(x;z) = P(y;z)$. (Symmetry essentially requires that the extent of measured poverty should be invariant with respect to a permutation of incomes across individuals: the 'names' of income recepients are irrelevant for the assessment of poverty).

Definition 2.2: Monotonicity (Axiom M). For all $z \in \mathbb{R}_{++}$ and all $x,y \in X$ such that x is obtained from y by an increase in the income of a poor person, $P(x;z) < P(y,z)$.
(Monotonicity requires the poverty measure to be negatively responsive to a poor person's income, other things being equal).

Definition 2.3: Respect for Income Dominance (Axiom D). For all $z \in \mathbb{R}_{++}$ and all $x,y \in X$ such that $n(x) = n(y)$ and $x^p G y^p$, $P(x;z) < P(y;z)$.

(Axiom D is borrowed from Amiel and Cowell (1994): it says that other things equal, poverty is less in x than in y if the vector of poor incomes in x dominates the vector of poor incomes in y).

Remark 2.4. Amiel and Cowell (1994) point out that Axioms M and D are independent, although they are frequently treated almost interchangeably in the literature; as it happens - again see Amiel and Cowell (op.cit.) - M and D are equivalent only if the poverty index also satisfies the symmetry axiom.

Definition 2.5: Transfer (Axiom T). For all $z \in R_{++}$ and all $x,y \in X$ such that $n(x)=n(y)$ and x^P is derived from y^P through a permissible progressive transfer, $P(x;z) < P(y;z)$.
(Axiom T is an equality-preferring property in a poverty index: it requires that ceteris paribus, a transfer from a poor person to a poorer person which does not render the former poorer than the latter should lead to a reduction in poverty).

Now, analogous to the transfer axiom for individuals, one could have an equality-promoting axiom for subgroups in poverty. The underlying motivation for such an axiom is well captured by Sen's (1973) Weak Equity Axiom which demands that an optimal distribution of income between two individuals would call for a transfer of income from the person with the higher level of welfare to the one with the lower level of welfare, provided that after the transfer the first person continues to enjoy at least as high a level of welfare as the second person. In this spirit, and as applied to subgroups in poverty, one has the following axiom of 'subgroup sensitivity'.

Definition 2.6: Subgroup Sensitivity (Axiom SS). For all $z \in R_{++}$ and all $x^P, x^N, y^P, y^N, x'^P, y'^P \in X$ satisfying

(i) $n(x^P)=n(x'^P) = n(y^P)=n(y'^P)$ and $x^N=y^N$;

(ii) $\mu^P(x^P,x^N,y^P,y^N;z) = \mu^P(x'^P,x^N,y'^P,y^N;z)$;

(iiia) $P(x^P,x^N;z) < P(y^P,y^N;z)$ and $P(x'^P,x^N;z) \leq P(y'^P,y^N;z)$; and

(iiib) $P(x^P,x^N;z) < P(x'^P,x^N;z)$ and $P(y^P,y^N;z) > P(y'^P,y^N;z)$, $P(x'^P,x^N,y'^P,y^N;z) < P(x^P,x^N,y^P,y^N;z)$.

(What Axiom SS says is the following. Let the poverty line be z. Suppose $a = (x^P,x^N)$ is the income vector of subgroup A and $b = (y^P,y^N)$ is the income vector of another subgroup B, with the vector of nonpoor incomes being the same in both a and b. Suppose further that through a pure redistribution of poor incomes within the vector $c = (a,b)$ we obtain the vector $c'=(a',b')$ where $a'=(x'^P,x^N)$ and $b'=(y'^P,y^N)$ in such a way that subgroup A's poverty level rises and subgroup B's poverty level declines without reversing the relative ranking of the poverty levels in the two subgroups. Then, Axiom SS requires that aggregate poverty for the vector c' be judged to be less than aggregate poverty for the vector c).

What is the class of poverty indices which satisfy the property of subgroup sensitivity in conjunction with some combination of other desirable properties discussed earlier? This question is addressed in the next section.

3. TWO IMPOSSIBILITY RESULTS

The following proposition is true.

Proposition 3.1. There exists no poverty index $P: X \times \mathbb{R}_{++} \rightarrow \mathbb{R}$ satisfying Axioms S, M and SS.

Proof. I shall assume the proposition to be false and derive a contradiction: to this end, a counterexample will suffice. Let $z=50$. Consider $x^P, x^N, y^P, y^N, x'^P, x'^N \in X$ such that $n(x^P)=n(x^N)=n(y^P) = n(y^N) = n(x'^P) = n(y'^P)=2$: this, notice, satisfies condition (i) in Definition 2.6 of Axiom SS. Suppose further that $x^P=(30,40)$; $x^N=y^N=(70,80)$; $y^P=(10,20)$; $x'^P=(20,40)$; and $y'^P=(10,30)$. Notice that $\mu^P(x^P,x^N,y^P,y^N;z)=\mu^P(x'^P,x^N,y'^P,y^N;z)$ (=25): so condition (ii) in Definition 2.6 is also satisfied. Recall, vide Remark 2.4, that since P has been assumed to satisfy Axioms S and M, P must satisfy Axiom D. Now, since $(y^P;y^N) = (10,20,70,80)$ is vector-dominated by $(x^P,x^N)=(30,40,70,80)$, by Axiom D, $P(x^P,x^N;z) < P(y^P,y^N;z)$; and since $(y'^P,y^N)=(10,30,70,80)$ is vector-dominated by $(x'^P,x^N)=(20,40,70,80)$, by Axiom D, $P(x'^P,x^N) < P(y'^P,y^N)$ - so that condition (iiia) of Definition 2.6 is satisfied. Next, since $(x'^P,x^N)=(20,40,70,80)$ is vector-dominated by $(x^P,x^N)=(30,40,70,80)$, Axiom D dictates that $P(x^P,x^N;z) < P(x'^P,x^N;z)$; and since $(y^P,y^N)=(10,20,70,80)$ is vector-dominated by $(y'^P,y^N)=(10,30,70,80)$, again by Axiom D one has $P(y'^P,y^N;z) < P(y^P,y^N;z)$ - so that condition (iiib) in Definition 2.6 is also satisfied. Briefly, all the antecedents in the statement of the subgroup sensitivity axiom are met. By virute of Axiom SS, we now have:

$$P(x'^P,x^N,y'^P,y^N;z) < P(x^P,x^N,y^P,y^N;z). \quad (3.1)$$

Notice, however, that $(x'^P, x^N, y'^P, y^N) = (20,40,70,80,10,30,70,80)$ is obtained as a permutation of $(x^P, x^N, y^P, y^N) = (30,40,70,80,10,20,70,80)$: when the person with income 20 exchanges his income with the person with income 30 in the vector (x^P, x^N, y^P, y^N), we obtain the vector (x'^P, x^N, y'^P, y^N).
By Axiom S, it must be the case that

$$P(x'^P, x^N, y'^P, y^N; z) = P(x^P, x^N, y^P, y^N; z). \qquad (3.2)$$

(3.1) and (3.2) are mutually incompatible, and this completes the proof of the proposition. (Q.E.D.).

The next proposition is also true.

Proposition 3.2. There exists no poverty index $P: X \times \mathbb{R}_{++} \to \mathbb{R}$ satisfying Axioms D,T and SS.

Proof. Again, we have a simple proof by contradiction. Let $z=100$. Consider $x^P, x^N, y^P, y^N, x'^P, y'^P \in X$ such that $n(x^P) = n(x^N) = n(y^P) = n(y^N) = n(x'^P) = n(y'^P) = 2$ (this is compatible with condition (i) in Definition 2.6 of the subgroup sensitivity axiom). Let it be the case that $x^P=(40,90)$; $x^N=y^N=(110,120)$; $y^P=(10,50)$; $x'^P=(20,90)$; and $y'^P=(10,70)$. It is easily verified that $\mu^P(x^P,x^N,y^P,y^N;z) = \mu^P(x'^P,x^N,y'^P,y^N;z)$ (=108.67) - so condition (ii) in Definition 2.6 is satisfied. Since $(x^P,x^N) = (40,90,110,120)\ G\ (10,50,110,120) = (y^P,y^N)$, $P(x^P,x^N;z) < P(y^P,y^N;z)$ by Axiom D; and since $(x'^P,x^N) = (20,90,110,170)\ G\ (10,70,110,120) = (y'^P,y^N)$, again by Axiom D , $P(x'^P,x^N;z) < P(y'^P,y^N;z)$ - so condition (iiia) in Definition 2.6 is satisfied. Also, since $(x^P,x^N)=(40,90,110,120)G(20,90,110,120)=(x'^P,x^N)$, Axiom D requires that $P(x^P,x^N;z)<P(x'^P;x^N;z)$; and $P(y'^P,y^N;z) < P(y^P,y^N;z)$ is dictated by Axiom D from the fact that $(y'^P,y^N)=(10,70,110,120)\ G\ (10,50,110,120) = (y^P,y^N)$: so condition (iiib) of Definition 2.6 is satisfied. The antecedents in the definition of the subgroup sensitivity axiom are all, thus, satisfied by the specifics of the present example. Axiom SS will then dictate that

$$P(x'^P, x^N, y'^P, y^N; z) < P(x^P, x^N, y^P, y^N; z). \quad (3.3)$$

But notice now that $(x^P, x^N, y^P, y^N) = (40,90,110,120,10,50,110,120)$ is obtained from $(x'^P, x^N, y'^P, y^N) = (20,90,110,120,10,70,110,120)$ through a permissible progressive transfer - of 20 from the person with income 70 to the person with income 20. Axiom T will then require that

$$P(x^P, x^N, y^P, y^N; z) < P(x'^P, x^N, y'^P, y^N; z). \quad (3.4)$$

From (3.3) and (3.4) we obtain a straightforward contradiction, and this completes the proof of the proposition. (Q.E.D.).

4. DISCUSSION

The two propositions proved in Section 3, simple as they are, serve to cast doubt on the universal acceptability of axioms like Symmetry and Transfer which for long have been considered to be fundamentally desirable properties for a poverty measure to satisfy. Closer examination suggests that both Symmetry and Transfer, which seem to conform naturally to any conception of justice in an 'individualistic' setting, are deeply inimical to the claims of 'group' justice. (This is very well brought out by Podder (1994) in the context of a discussion of the notions of relative deprivation and inequality).

In fact, it would appear that the notion of 'reverse discrimination' is unsustainable without violence to Symmetry. It is useful, in this context, to recall the distinction which Dworkin (1977) makes between two types of the right to equality which he calls, respectively, ' the right to equal treatment' and 'the right to treatment as an equal'. The former is '... the right to an equal distribution of some opportunity or resource or burden', while the latter is '... the right, not to receive the same distribution of burden or benefit, but to be treated with the same respect or concern as anyone else' (Dworkin 1977, p.227). Further, '... the right to treatment as an equal is fundamental, and the right to equal treatment, derivative. In some circumstances the right to treatment as an equal will entail a right to equal treatment, but not, by any means, in all circumstances' What Axiom SS does is to dismiss the claim to equal treatment for individuals drawn from subgroups that, poverty-wise, stand in unequal relation to each other. Similarly, in recognition of the claims of substantive over formal justice - claims which must take account of the precise affiliation of different persons to different groups - Axiom SS would sometimes penalize a progressive transfer of income from a poor individual belonging to a relatively badly-off group to a poorer individual belonging to a relatively better-off group—leading to a violation of Axiom T. (Care would have to be taken, of course, to ensure that the way in which society is partitioned into groups is consistent with a sensible and meaningful classification that takes account of the relevant economic, social and political facts governing the society under review). The point is that if a poverty index is at all to be sensitive to the issue of intergroup unbiasedness in the distribution of poverty, then certain criteria that have been advanced as virtually indispensable for an 'acceptable' poverty measure may have to be seriously reconsidered.

It is worth emphasizing that the subgroup sensitivity axiom focusses attention on a phenomenon which is a pervasive feature of social and economic life in many societies, but does not seem to have been adequately taken into account in the formal literature on the measurement of poverty: the phenomenon in question is that of discrimination. Thurow has written compellingly on the subject, and the following extracts from his book (Thurow, 1981; pp.178,179,180,182) are of particular relevance to some of the issues raised in this note:

> Western economics is at its heart an economics of the individual ... Group welfare is, if anything, only the algebraic summation of the individual welfare of the members of the group... Is the correct economic strategy to resist group welfare measures and group redistribution programmes wherever possible? Or do groups have a role to play in economic justice? ... Within any group - no matter how privileged - there will be individuals who have been denied equal opportunities and suffered from discrimination, but they have not been subject to a systematic denial of opportunities ... Conversely, within any group - no matter how underprivileged - there will be individuals who have not suffered from a systematic denial of opportunities... Discrimination affects individuals, but it can only be identified at the level of the group. As a result, it is not possible for society to determine whether it is or is not an equal opportunity society without collecting and analyzing economic data on groups ... Individuals have to be judged based on group data, yet all systems of grouping will result in the unfair treatment of some individuals... [E]very society has to have a theory of legitimate and illegitimate groups and a theory of when individuals can be judged on group data and when they cannot be judged on group data. A concern for groups is unavoidable.

But a concern for groups, of the type captured by Axiom SS, is not accommodated by certain restrictions that are conventionally imposed on a poverty measure.

5. CONCLUDING OBSERVATIONS

In the end, one has to agree with Sen (1981, p.194) when he says, in the context of the Sen index of poverty P^S: 'While [P^S] has certain unique advantages, which its axiomatization brings out, several of the variants are certainly permissible interpretations of the common conception of poverty. There is nothing defeatist or astonishing in the acceptance of this 'pluralism'. Indeed, ... such pluralism is inherent in the exercise'.

Allowing for pluralism may also entail accomodating tradeoffs between alternative principles governing the measurement of poverty. In the present context, for example, the subgroup sensitivity axiom might be regarded as being excessively demanding: income transfers among the poor, however progressive, may be proscribed by the axiom. By the same token, the transfer axiom could also be seen as being overly restrictive: <u>any</u> progressive income transfer among the poor would have to be approved, even if it exacerbates between-group poverty differences. The object of this paper has not been to urge an unqualified acceptance of the subgroup sensitivity axiom, but only to point out - in the course of challenging some aspects of prevailing orthodoxy - that Symmetry and Transfer may be overpraised virtues. Under these circumstances, a sensible approach might be to look for poverty indices which are not of the 'all-or-nothing' variety - indices which allow for some role to be played by the transfer axiom and some role to be played by the subgroup sensitivity axiom: neither axiom is thus wholly abandoned in the cause of the other, nor wholly endorsed to the exclusion of the other. (Some preliminary results on the derivation of such 'compromise' poverty indices are available with the present author). Sen points the way to these considerations in the poverty-measurement context when he alludes, in the course of a discussion of impossibility results in social choice theory, to '...the serverness of the problem of postulating absolute principles of collective choice that are supposed to hold in every situation' (Sen, 1970; p.178).

To take a liberty with Wittgenstein, the meaning of a poverty index is its use. And if a poverty index can be devised which is useful for assessing how fairly or otherwise the burden of poverty is distributed across different subgroups in a society, then such an index must be regarded as being meaningful. This is so even if, as a consequence, certain axioms of poverty measurement - however sacred in other contexts and for other purposes - have to be laid to rest, or at least have to have their purity somewhat sullied.

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